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EFFECT OF AN INFLECTION IN THE PROFILE OF MEAN VELOCITY ON THE RESONANCE INTERACTION OF PERTURBATIONS IN A BOUNDARY LAYER

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The character of the laminar-turbulent transition (LTT) in shear flows depends to a considerable extent on the distribution of the vorticity of the average motion. According to the linear theory of stability, the appearance of extrema in such distributions (points of inflection in the velocity profile) leads to expansion of the spectrum and an increase in the increments of unstable pulsations that are already taking place (see [1, 2]). Both the time of formation of the nonlinear regime and the character of its occurrence are variable.

The appearance of inflections may be due either to external flow conditions or to the nonlinear self-perturbation of "primary" waves in the flow. Examples of the effect of such mean flow singularities on the interaction of wave perturbations were examined in [3] for free shear layers and in [4] for pre-separation boundary layers. However, the laws governing the evolution of interacting waves under these conditions have yet to be definitively established.

The goal of the present investigation is to explore features of the effect of the characteristics of inflected profiles on resonance wave interactions in boundary layers. The results that are obtained are used to interpret the mechanism responsible for preventing the occurrence of a subharmonic S-type transition with an increase in the level of the initial perturbations.

We choose a flow with the profile $U_G(y)$ [5] as the initial flow for studying the evolution of resonance perturbations. This flow models the motion of intensive eddies in a boundary layer:

$$U_G(y) = U_\pm + \varkappa(\mathrm{th}\,(y - y_r)/\delta \mp 1),\ y \gtrless y_r.$$

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Here, y_r determines the position of the point of inflection relative to the wall; $\varkappa$ and $\delta \ll 1$ characterize the magnitude and width of the inflection region; $U_{\pm}$ corresponds to the solutions of the Blausius equation; $U_{\pm}(y) = U_B(y)$ corresponds to the Blausius profile with $\varkappa = 0$. The choice of U_G is dictated foremost by the simplicity of its connection with the parameters of the inflection.

Remaining within the framework of the approximation of weakly linear theory, we represent the flow field in the form $\mathbf{U} = (U(y), 0, 0) + \varepsilon(u_1, u_2, u_3) + O(\mathrm{Re}^{-1})$, where quasi-harmonic perturbations with $\varepsilon \ll 1$ can be written

$$u_j = \sum_k A_k \psi_{jk} e^{i\theta_k}, \quad j = 1, 2, 3,$$

$\psi_{jk}(y)$ and $\theta_k = \int \alpha_k dx + \beta_k z - \omega(\alpha_k, \beta_k) - i\gamma(\alpha_k, \beta_k)$ are determined from the solution of the Orr–Sommerfeld (OS) equation. In the first nonlinear approximation [6, 7], three-wave resonance interaction plays the leading role in the formation of the S-regime of the LTT in a Blausius boundary layer (U_B). Here, the most intensive interaction takes place in symmetrical triads (k = 1, 2, 3) including a plane wave (α_1, $\beta_1 = 0$, ω_1) and a pair of oblique Tollmien–Schlichting (TS) waves ($\alpha_{2,3} \approx \alpha_1/2$, $\beta_2 = -\beta_3$, $\omega_{2,3} = \omega_1/2$) [6, 7]. The behavior of the amplitudes $B_k = A_k \exp(\gamma_k t)$ in such a triad conforms to the system

$$\left(v_1 \frac{\partial}{\partial x} - \gamma_1\right) B_1 = S_1 B_2 B_3 h(-\Delta),$$
$$\left(v \frac{\partial}{\partial x} - \gamma\right) B = S B_1 B_{3,2} h(\Delta), \quad B_{k0} = B_k(x_0). \qquad (1.1)$$

Here, $\gamma = \gamma_{2,3}$; v_1, $v = v_{2,3}$, S and S_1 are expressed through eigenfunctions of the direct and conjugate OS problems, while

$$h(\Delta) = \frac{1}{X} \int_0^X dx \exp\left(i \int_0^x \Delta\, dx'\right), \quad \Delta = \alpha_1 - 2\alpha_{2,3}, \quad 1 \ll \alpha X \ll \varepsilon^{-1}$$

(X is the averaging interval).

According to calculations, the pattern of evolution of the triad at $\varkappa > 0$ qualitatively reproduces the features of its motion in the Blausius flow. A section in which subharmonic spaces waves $|B_{2,3}| < |B_1|$ undergo parametric pumping is followed by explosive development of the nonlinear stage at $|B_{2,3}| \geqslant |B_1|$ (Fig. 1). The appearance and subsequent development of the inflection ($\varkappa > 0$) may facilitate intensification of the subharmonic pumping, mainly as a result of an increase in γ_1 and γ. As was shown in [2], such an increase occurs at values of y_r belonging to a certain finite interval ($0.7 \leqslant y_r \leqslant 2.5$) within ($0 \leqslant y < 5$) the boundary layer. An increase in $\varkappa$ also promotes a slight increase in [S]. Within the range of parameters investigated here, the dependence of S on ω and Re turns out to be slight. Lines 1-3 in Fig. 1 show values of $B_1(x)$ and $B_{2,3} = B(x)$ for $F_1 = 2F_2 = 115 \cdot 10^{-6}$, $\beta_2/\alpha_2 = 1$ at $\varkappa = 0$ (undeformed boundary layer U_B); $y_r = 2$, $\varkappa = 2$ %; $y_r = 1$, $\varkappa = 2$ %.

The angles of propagation $\xi = \arctan \beta/\alpha$ have a greater effect on the efficiency of the interaction. The solid lines in Fig. 2 show the dependence of the pumping increments $\sigma = \partial \ln|B|/\partial x$ on the parameter $\tan\xi$ at $y_r = 2$, $\varkappa = 0$; 10% (curves 1 and 2) for different pumping amplitudes $|B_{10}|$. The dashed lines, corresponding to the position $\max_\xi \sigma(B_{10})$, shift to the region of greater ξ with an increase in $\varkappa$ and become the asymptote $\xi_m \approx 71°$ at $\varkappa \geqslant 10\%$. Thus, the presence of the inflection promotes more efficient excitation of the perturbation field in three dimensions, since at $\varkappa = 0$ the maximum increments correspond to $\xi_m \leqslant 60°$. The universality of this statement is violated at y_r near the critical layer of the wave $y_c \approx 1$. The relation $\xi_m(\varkappa)$ turns out to be nonmonotonic: at $0 \leqslant \varkappa \leqslant 2\%$, there is a decrease in ξ_m with an increase in $\varkappa$, i.e., the configuration of the wave vector in the triad of maximally interacting waves becomes "flatter" compared to the boundary layer U_B. With a further increase in the size of the inflection $\varkappa$, the angle ξ_m becomes the asymptote $\xi_m \approx 71°$. Curves 1 and 2 in Fig. 3 show values of $\tan\xi_m(\varkappa)$ with Re = 625, $F_1 = 115 \cdot 10^{-6}$ and $y_r = 1, 2$.

The above-examined features of the development of resonance triads concern a special type of flow U_G, and their generality needs to be discussed. To this end, we examined flows with profiles approximating motion in the pre-separation zone of a boundary layer $U_S(y)$ [4] and flows seen in the transitional region with the LTT K-regime $U_K(y)$ [8]. Here, as the parameter $\varkappa$ (degree of inflection) we took the maximum deviation of these profiles from the

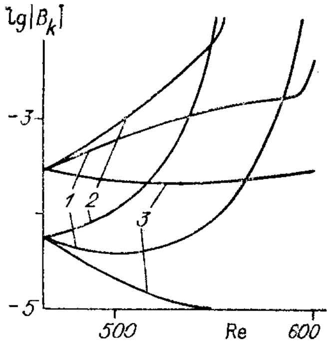

Fig. 1

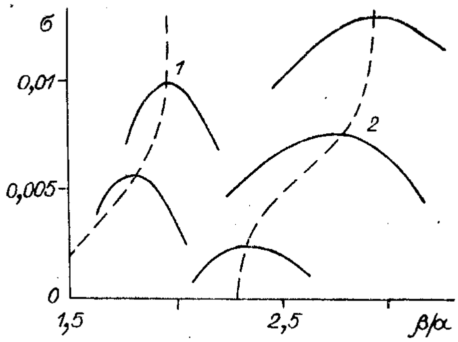

Fig. 2

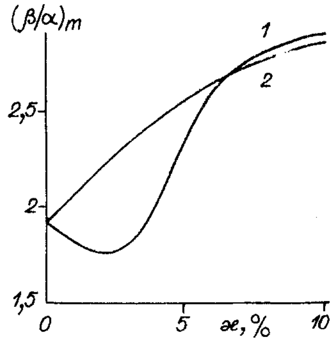

Fig. 3

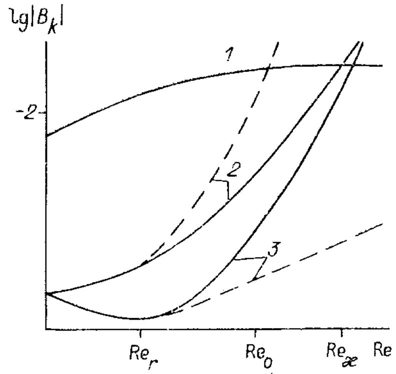

Fig. 4

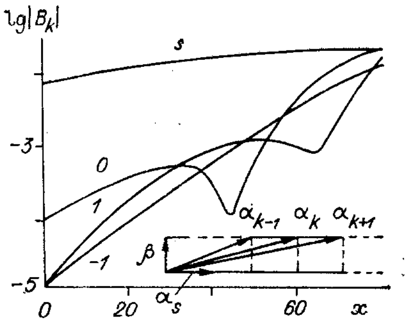

Fig. 5

distribution $U_B(y)$. The characteristics of the evolution of perturbations in the flow with U_G are qualitatively reproduced in the flow U_K: still valid are the intensification of subharmonic pumping in the triads and an increase in the limiting angles ξ_m ($\xi_m \to 70°$). In contrast to the cases U_G and U_K, $\xi_m \approx$ const in flows with reverse motion (the profile U_S). The latter finding is consistent with the results obtained in [4]. The transitional zone of the boundary layer on a smooth plate is characterized by an absence of reverse flows. The phenomena noted for U_G are typical.

However, in contrast to the Blausius flow, the efficiency of parametric resonance of the low-frequency (LF) oscillations does not guarantee their leading position in the spectrum (S-regime of LTT), since the inflection in the profile U(y) simultaneously sharply intensifies linear instability in the high-frequency (HF) region. It is obvious only that the parallel occurrence of these processes wil help accelerate the transition to turbulent flow.

In the above study, we examined the interaction of "secondary" perturbations, i.e., perturbations that evolve after the formation of the inflection in the profile. Additional singularities may arise in the case of a profile inflection caused by the self-perturbation of an intensive Tollmien–Schlichting "primary" wave (α_s, $\omega_s + i\gamma_s$). Such an occurrence might be due to the formation of a nonlinear critical layer at $y_r \approx y_c$ and, according to [9-11], might be accompanied by stabilization of the oscillations ($\gamma_s \to 0$) with no change in the form of the velocity profile and with a dispersion coupling that is close to linear. Under these conditions, the dominant mechanism of interaction might turn out to be resonance of the primary wave with background pulsations generated in accordance with the distorted mean profile.

Staying within the framework of the above representations, we analyzed the efficiency of the parametric interaction of pairs of secondary LF perturbations (calculated for the profile U_G with $y_r \approx y_c$ and $\varkappa \geqslant 0$) in the field of a primary wave with a TS distribution. The equations which describe such a system differ from (1.1) in the values of the coefficients (the index s corresponds to a wave with k = 1). Calculations of S showed that the increment of parametric pumping $\sigma_p \sim |SB_{10}|$ under these conditions is close to the case of motion in a flow with $\varkappa = 0$ and is markedly lower than in triads of "secondary" oscillations. On the other hand, the distribution of $\max \sigma_p(\xi)$ nearly coincides with the latter case. As a result, there is a possibility of a relative slowing of the growth of LF pulsations in the field of the "primary" TS wave. The total increment $\sigma = \gamma + \sigma_p$ decreases due to stabilization of the linear instability (a decrease in γ). At $\varkappa \leqslant 2$ % $(y_r \approx y_c)$, this is a direct consequence of the inflection [2], while at $\varkappa > 2\%$ it is the result of the rotation of ξ in the region of more oblique waves — larger values of β. Thus, the development of a nonlinear critical layer may be associated with the appearance of a mechanism which prevents the intensive excitation of LF oscillations through a decrease in γ_1 and γ. The effectiveness of this mechanism can be evaluated on the basis of the relations $\tau_N = |B_{10}|^{-1/2}$ and $\tau = |\gamma + SB_{10}|^{-1}$ — the characteristic times of the nonlinear critical layer and parametric amplification in the triad. Satisfaction of the inequality $\tau_N/\tau \lesssim 1$ establishes the upper bound of the quantity $|B_{10}|$ for occurrence of an S-transition. Figure 4 offers a qualitative representation of the development of the triad (curves 1-3, respectively, show the amplitudes of a two-dimensional primary wave B_1 and three-dimensional subharmonic secondary waves $B_{2,3}$ with $\beta/\alpha \approx 2$ and 3; the dashed lines show the behavior of the subharmonics on the undeformed mean flow U_B). If an inflection $\varkappa > 0$ is formed in a certain section Re_r of the boundary layer as a result of the development of a nonlinear critical layer, then the rate of growth of subharmonics with different values of β/α will change (solid lines 2 and 3). Thus, the coordinate Re_0, where $|B_{2,3}(x)| = |B_1(x)|$, is displaced downstream toward $Re_\varkappa$.

It is interesting that a TS primary wave has a direct effect on the development of HF pulsations — the linear instability of which is intensified considerably with an inflection U(y) within a broad spectral band. To analyze this phenomenon, we examined the resonance interaction of such "secondary" HF pulsations with $y_r = y_c$, $\varkappa > 0$ in the field of a TS wave (ω_s, α_s, $\beta_s = 0$). Perturbations of the HF spectrum were approximated by a discrete set of harmonics with (α_k, β), $\omega_k + i\gamma_k$, $\omega_{k\pm 1} = \omega_k \pm \omega_s$, belonging to the neighborhood $\omega_0(\varkappa)$ with $\gamma_0 = \max \gamma_k$. The system of amplitude equations takes the form

$$v_k \frac{dB_k}{dx} = \gamma_k B_k + S_k^- B_s B_{k-1} h(-\Delta_{k-1}) + S_k^+ B_s^* B_{k+1} h(\Delta_k),$$
$$\Delta_k = \alpha_{k+1} - \alpha_k - \alpha_s, \ -N \leqslant k \leqslant N.$$

It is evident from the calculations that the coefficients $S_k^\pm$ increase with an increase in frequency ω_k and the parameters β and $\varkappa$.

Figure 5 shows graphs of the amplitude relation $B_k = B_k(x)$ with Re = 1040, N = 5, $y_r = 2$, $\varkappa = 2\%$, $\beta = 0$, $\omega_s = 40 \cdot 10^{-6}$, $\omega_0 = 400 \cdot 10^{-6}$. The numbers of the curves correspond to the k-th harmonic. The interaction has almost no effect on the rate of amplification of HF oscillations ($\sigma_k \approx \gamma_k$) and reduces to an exchange of energy between them. The latter is manifest in a rapid equalization of the amplitudes B_k of a perturbation during the initial stage and their oscillation during development of the process.

Thus, the appearance of an inflection in the mean-velocity profile of a boundary layer due to the development of an intensive TS wave creates the conditions necessary for slowing the growth of LF oscillations and the formation of subharmonics. At the same time, intensification of three-dimensional oscillations in the HF part of the spectrum is facilitated. The study conducted here is of a qualitative nature, but the conclusions that were reached make it possible to discern the mechanism responsible for the disappearnce of the S-regime and the formation of the "high-frequency" types of LTT's seen in a boundary layer with an increase in the level of an induced wave [8].

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LONGITUDINAL VORTEX STRUCTURES AND HEAT TRANSFER IN THE REGION OF ATTACHMENT OF A SUPERSONIC TURBULENT BOUNDARY LAYER

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The formation of longitudinal vortex structures in boundary layers of supersonic flows has been observed in experiments conducted by several authors [1-7] who have studied the development of the separation region in plane, axisymmetric, internal, and external flows moving past a body. Both laminar and turbulent flow regimes have been studied in this regard. The development of structures usually termed Taylor–Görtler (T–G) vortices in the neighborhood of the point of attachment in flows after a projection or after a separation point in constricted regions leads to a strictly ordered redistribution of processes involving heat and momentum transfer and their periodic change in the direction transverse to the flow. Under conditions whereby heat transfer is intensified in attachment regions due to an increase in the level of turbulent pulsations [8], the development of secondary flows can lead to additional thermal loads in the regions of their peak values.

From the viewpoint of the development of natural (internal) instabilities in a system, resulting in adaptation, the appearance of T–G vortices is one link in a chain of hierarchical changes in the structure of a boundary layer. It is also significant that the result of loss of stability in the system – the creation of stationary vortices – can be stored in the "memory" of the flow far downstream from the immediate source of the instability.

In the present investigation, we discuss experimental studies of three-dimensional features of flow and heat transfer due to T–G vortices. A second boundary layer of longitudinal structures is observed, the mechanism of its formation not being connected with vortices of the T–G type.

Experimental Conditions. Measurements of pressure field and heat transfer on models of steps were conducted in the T-333 wind tunnel at the Institute of Theoretical and Applied Mechanics (of the Siberian Branch of the Soviet Academy of Sciences) with a jet 304 mm in diameter. The experiments were performed inside an Eifel chamber with the Mach numbers M_1 = 2.0, 3.0, 4.0, and 5.0 for the incoming flow. The range of the Reynolds numbers $Re_1 = (30\text{-}100)\cdot 10^6\ m^{-1}$, while the range of stagnation pressures $p^* = 180\text{-}1600$ kPa. Stagnation temperature T^* varied within the range 260-270 K.

The height of the step (Fig. 1a) located on the plate of width b = 120 mm was h = 6.0, 6.4, and 15.0 mm. The angle of inclination of the face of the step β = 90, 25, and 65°, respectively. The distance from the leading edge of the plate to the vertex of the angle of convergence was 177 mm. We glued a 4-mm-wide vortex generator to the plate 6 mm from its leading edge. The height of the sandy roughness of the generator was 0.2 mm. The characteristic thickness of the boundary layer in the undisturbed flow ahead of the interaction region $\delta_1 \approx 2.1$ mm. Limiter plates (flanges) 30 mm high were installed on the lateral walls of the model to prevent flows in the transverse direction.

Measurements of the local heat-transfer coefficients were made by a modification of the electrocalorimetric method [9] in a complex which included an automatic data processing system

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